Prediction Model to design Infrared Lamp Array Setups

Herbert Paula de Vasconcelos  
Mechanical Engineering Division  
Instituto Tecnológico de Aeronáutica  
Bolsista PIBIC-CNPq  
herbertpv@hotmail.com

Ezio Castejon Garcia  
Mechanical Engineering Division  
Instituto Tecnológico de Aeronáutica  
ezio@ita.br

Abstract: The heat flux analytical modeling from an infrared lamp is very difficult. These lamps have a quartz involucres and an internal reflective surface to converge the emitted radiation to a determined area. These empiric parameters influence the heat flux evaluation and turn this study to experimental analysis, in order to complete the analytical. The objective of this paper is to obtain a computational program to simulate the heat flux from an infrared lamp array, using a modeling analytical-experimental. An equation to simulate the heat flux from a lamp was obtained beyond an analysis of the radiation properties, geometrical lamp parameters and empirical equations. Using the electromagnetic wave theory in the particular case of radiation from an infrared lamp, it was get one of the terms of the equation, which is only function of the geometrical lamp parameters. The other term is empirical and was obtained from analyze of experimental data. The complete equation was used to implement the program.

Keywords: tungsten filament lamp, heat flux, mathematical model, heat transfer, radiation.

1. Introduction

This paper has as objective the study of the heat flux from a tungsten filament lamp. The study defines an equation that describes the amount of heat that arrives in a determined point of a given surface, which receives the radiation from a lamp. Once, with the fulfilled development, this technique of heating can be used in many applications, as example for Satellite Space Simulation.

2. Tungsten Filament Lamp

The principal objective of this paper is to obtain an equation to give the incident radiation from a tungsten filament lamp in a point. There are many kinds of infrared lamps and for each one; the manufacturing process has influence in the radiation properties. The selected lamp was a model, produced by Research INC, which is illustrated in Fig. (1).

Figure 1: The model of studied infrared lamp [1].

The specified lamp has a golden reflector, in a parabolic format, in order to get better the efficiency of emitted radiation from the filament. This reflector has a high influence in the results, making more difficult the analysis of the heat flux from the lamp.

Specification of the lamp, used in the experimental test, is as follows:
Type
Overall Length
Lighted Length
Rated Voltage
Current at Rated Voltage
Total Power Dissipated at Rated Voltage
Average Life
Color Temperature
Possible Corona Region in Dry Air
Brightness
Usual Size, Inches (mm)
Usual Range of Peak Energy Wavelength
Radiation
Relative Response to Heat-up
Relative Response to Cool-down
Mechanical Shock
Thermal Shock

500T3/CL Research Inc, Tungsten Filament Wire, T3 Quartz Lamp
224 mm – 8.81 inches
127 (mm) – 5 inches
120 V
4.17 Amps
5000 W
5000 hours
2500 K
None
Bright White
0,375 or Dia. Tube (9,525)
0.89 to 1.5 Microns
72 to 86 %
Seconds
Seconds
Good
Excellent

Figure (2) shows a graphic, which the directional response heat flux from the lamp is modeled as proportional to the inverse of the squared distance \(d^2\) [2]. The values in the y-axis are relative to the parameter \(R\), Eq. (1), and \(E\) is the heat flux. X-axis is referent to the solid angle of a specified point.

\[
E = \frac{R}{d^2}
\]  

(1)

Figure 2: Behavior of heat flux from an infrared lamp, obtained by the Simulate IR Series manufacturer [2].

3. Heat Flux Model

3.1 Radiation

Figure (3) shows the definition of the solid angle. To evaluate the radiation \((dE)\) that leaves a specified surface, in a determined direction, it is necessary to multiply its intensity by the projection surface in the chosen direction.
The value of the radiation $dE_n$, cited above, could be evaluated as described in Eq. (2) [3]:

$$dE_n = I_n \cdot \cos \theta \cdot d\omega$$  \hspace{1cm} (2)

Where:
- $I_n$: black body intensity;
- $d\omega$: solid angle element.

Then, the heat flux that leaves the surface and crosses a semi hemisphere (SH) is given by Eq. (3):

$$E_n = \int_{SH} I_n \cdot \cos \theta \cdot d\omega$$  \hspace{1cm} (3)

Figure (3) presents the solid angle element that could be expressed in terms of the angles $\theta$ e $\phi$ of a spherical coordinates system, which is centered in a given filament point. Just remember, that the solid angle element is obtained dividing the area in the hemisphere by the square of the hemisphere radius. Then:

$$d\omega = \text{sen}\theta \cdot d\theta \cdot d\phi$$  \hspace{1cm} (4)

In consequence, integrating Eq. (3) and utilizing the relation given by Eq. (4):

$$E_n = \int_{0}^{\frac{\pi}{2}} \int_{0}^{\frac{\pi}{2}} I_n \cdot \cos \theta \cdot \text{sen}\theta \cdot d\theta \cdot d\phi$$  \hspace{1cm} (5)

There are some cases that the radiation intensity doesn’t have dependence from the direction (diffuse):

$$E_n = \pi I_n$$  \hspace{1cm} (6)

### 3.2 Heat Flux from one Infrared Lamp

The lamp length $L$ is considered parallel to the $xy$ plan and the height above the origin is $H$. The angles $\theta$ e $\phi$ defines the longitudinal and radial directions, respectively. The lamp area element is represented by $dA$, and for irradiated surface, it is $dA$. The angle $\gamma$ is defined between the normal vector $N$ and $D$ vector, from surface to lamp. The angle between $yz$ plan and $D$ is represented by $\psi$ [4].
Figure 4: Lamp geometrical description [4].

The radiation intensity, $j$ [Watts], that leaves the surface $dA_L$ in $D$ direction is, [2]:

$$j = \frac{E}{\pi} \cos \psi \cdot f(\phi) \cdot dA_L$$  \hspace{1cm} (7)

Where:
- $E$: lamp power emission;
- $f(\phi)$: reflector influence.

The amount of radiation that arrives in surface $dA$ is proportional to the solid angle [5]:

$$d\omega = \cos \gamma \cdot \frac{dA}{D^2}$$  \hspace{1cm} (8)

So, the incident radiation $dq$ received by $dA$ is:

$$dq = \int j \cdot d\omega$$  \hspace{1cm} (9)

Using Eq. (7) and Eq. (8) in Eq. (9), the incident radiation is given by Eq. (10) below:

$$dq = \int_{A_L} \frac{E}{\pi} \cos \psi \cdot f(\phi) \cos \gamma \cdot \frac{dA}{D^2} \cdot dA_L$$  \hspace{1cm} (10)

The lamp area element $dA_L$ could be replaced by $W \cdot dl$. Squared distance is given by $D^2 = (x-l)^2 + y^2 + H^2$. Replacing $\psi$ e $\gamma$ in terms of $x$, $y$, $z$ e $l$, Eq. (11) and Eq. (12) and using the cited relations, Eq. (10) becomes in this format:

$$\cos \psi = \frac{r}{D}$$  \hspace{1cm} (11)

$$\cos \gamma = \frac{H}{D}$$  \hspace{1cm} (12)
\[
\frac{dq}{dA} = \int_{\lambda} E \cdot f(\phi) \cdot \frac{r}{D} \cdot \frac{H}{W} \cdot \frac{dl}{D^2}
\]

\[
\frac{dq}{dA} = \frac{EW}{2\pi} \cdot f(\phi) \int_{-L/2}^{L/2} \frac{2rH}{D^4} \cdot dl
\]

\[
I = \frac{dq}{dA} = K(V) \cdot f(\phi) \cdot 2Hr \int_{-L/2}^{L/2} \frac{dl}{\left[(x-l)^2 + r^2\right]^{3/2}}
\]

Where:
\[
r^2 = y^2 + H^2
\]
\[
K(V) = EW/2\pi, \text{ is a lamp voltage function.}
\]

Defining:
\[
\delta_R = L/2 - x
\]
\[
\delta_L = -L/2 - x
\]

Equation (11) could be integrated and its final expression is:

\[
I = K(V) \cdot f(\phi) \cdot \left(\frac{H}{r}\right) \left[\frac{\delta_R}{\delta^2_x + r^2} - \frac{\delta_L}{\delta^2_L + r^2} + \frac{1}{r} \left(\tan^{-1} \frac{\delta_R}{r} - \tan^{-1} \frac{\delta_L}{r}\right)\right]
\]

\[
I = [W/m^2]
\]

The above equation has three empirical parameters: \(L, f(\phi), K(V)\). The lamp length L must be determined from the variation of the intensity in X-axis, considering the end effects. The reflector function \(f(\phi)\) could be obtained from the variation of the intensity in Y-axis. The function \(K(V)\) is obtained from the calibration of the lamp for different values of the voltage.

4. Results

4.1 Empirical Parameters

To obtain the final equation to calculate the heat flux from an infrared lamp, it is necessary to determine the two empirical functions present in the Eq. (18), described below as:

\(K(V)\): function of the applied voltage in the lamp;
\(f(\phi)\): function of the angle \(\phi\).

In accordance to Fig. (4), angle \(\phi\) can be defined as:

\[
\phi = \arctan \left(\frac{y}{H}\right)
\]

Such above equations cannot be analytically obtained. First, it is necessary to obtain experimental data from one lamp set-up test. Then, applying numerical method, those functions are obtained, and the intensity from Eq. (18) can be finally calculated.
4.1.1 Experimental Analysis

The data was obtained from fulfilled experimental tests at Integration and Testing Laboratory (LIT) of the Instituto Nacional de Pesquisas Espaciais (INPE), in the Thermal Vacuum Test Division [1].

4.1.2 Experimental apparatus

The experimental apparatus has three essential elements: tungsten filament lamp (Infrared Heaters, model 5236, 500W of maximum power), plate of radiometers and connections.

The plate, in aluminum, is a quadrangular base with thermal blanket isolation (MLI - multilayer isolation). Over of this, 49 quadrangular copper radiometers were installed. Each radiometer is a cooper plate, black painted in its external surface. Thermocouples were installed in radiometer intern sides, to get temperature measurements. The radiometers were thermal isolated from aluminum plate by nylon wires.

The lamp support was fixed in central position with its arm assembled in the plate extremity, as show in Fig. (5). This apparatus permits modification in any position of the lamp in the xy plan, as well, its height \( H \).

![Figure 5: Experimental apparatus [1].](image)

4.1.3 Experimental description

The test has been divided in two parts. In the first one, the lamp has been located in the center of the plan \( xy \), with its height varying among five values and the power applied fixed in 500 W. In second, the lamp has been located in the extremity of the plate, height fixed in 35 cm and the power applied varying among five values. The experiment had as objective to evaluate the behavior of the heat flux in the plate related with the parameters: height and power.

4.1.4 Data analysis

The gotten data in the tests has relation to the heat flux, measured on the radiometers. The equation to give the heat flux intensity is function of some parameters: \( H \) (height), \( x \) and \( y \) (position of the lamp in the plan \( xy \)) and \( V \) (applied power).

The intensity has been considered in the center of each radiometer, relating the values of \( x \), \( y \), \( H \) and \( V \) for each heat flux. Such parameters are placed in the Eq. (18). In this way, information about the product of the two desired functions can be obtained.

The amount of gotten data is enormous, however a filtering was made to separate those most important. These data has been organized in two tables, each one for each test. The first one has not given cleared results, however of second one has been gotten graphics that show an idea of the function behaviors.

<table>
<thead>
<tr>
<th>( V (W) )</th>
<th>( \phi ) (degree)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>11,3</td>
</tr>
<tr>
<td>502</td>
<td>1,90</td>
</tr>
<tr>
<td>453</td>
<td>1,71</td>
</tr>
<tr>
<td>399</td>
<td>1,53</td>
</tr>
<tr>
<td>376</td>
<td>1,44</td>
</tr>
<tr>
<td>330</td>
<td>1,28</td>
</tr>
<tr>
<td>299</td>
<td>1,17</td>
</tr>
<tr>
<td>251</td>
<td>1,00</td>
</tr>
</tbody>
</table>

Table 1: Values of product \( K(V)f(\phi) \) as functions of the parameters \( V \) and \( \phi \).
4.1.5 Interpolation

It has been made an interpolation with the values of the product $K.f$ for the two cited cases above. For the function $K.f$, in relation to angle $\phi$, the function’s behavior is a polynomial, third order. And in relation to $V$, the product can be considered as a linear function. Then, it can be concluded that the two empirical functions can be described as:

$$K(V) = a + b.V$$  \hspace{1cm} (21)

$$f(\phi) = a_0 + a_1\phi + a_2\phi^2 + a_3\phi^3$$  \hspace{1cm} (22)

The founded coefficients in the interpolation are shown in the Tab. (2):
Table 3: Coefficients of the interpolation, considering $K.f$ as a polynomial of third order.

<table>
<thead>
<tr>
<th>$V$</th>
<th>$A_0$</th>
<th>$A_1$</th>
<th>$A_2$</th>
<th>$A_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>251</td>
<td>1.89</td>
<td>5.90E-03</td>
<td>7.85E-04</td>
<td>-1.55E-05</td>
</tr>
<tr>
<td>299</td>
<td>1.70</td>
<td>4.95E-03</td>
<td>7.49E-04</td>
<td>-1.49E-05</td>
</tr>
<tr>
<td>330</td>
<td>1.52</td>
<td>4.90E-03</td>
<td>6.42E-04</td>
<td>-1.29E-05</td>
</tr>
<tr>
<td>376</td>
<td>1.43</td>
<td>4.80E-03</td>
<td>6.12E-04</td>
<td>-1.24E-05</td>
</tr>
<tr>
<td>399</td>
<td>1.27</td>
<td>5.18E-03</td>
<td>4.95E-04</td>
<td>-1.03E-05</td>
</tr>
<tr>
<td>453</td>
<td>1.16</td>
<td>4.45E-03</td>
<td>4.68E-04</td>
<td>-9.59E-06</td>
</tr>
<tr>
<td>502</td>
<td>0.99</td>
<td>3.66E-03</td>
<td>3.91E-04</td>
<td>-7.90E-06</td>
</tr>
</tbody>
</table>

The coefficients, from Tab. (3), reference to estimate the real values. Considering the magnitude of the order of these values, the problem has been solved using the tool Solver in Microsoft Excel. The objective function is the correlation coefficient between experimental and the derived values by the method. This function has been maximized. The obtained results for the real coefficients are:

$$K(V) = 0.70784 + 0.00100.V$$  \hspace{1cm} (23)

$$f(\phi) = 1.48153 +0.00100. \phi + 0.00003.\phi^2 + (7.07E-07).\phi^3$$  \hspace{1cm} (24)

The correlation coefficient for this comparison was 0.18. In addition, these values are too far from ideal range, which is more than 0.8.

### 4.2 Heat Flux from an Infrared Lamp Array

After of the implementation of the steps above, the equation has all functions defined. Then, the next step is to extend the evaluation to a lamp array. To do this, it is necessary to define the coordinate system: $x,y$.

First, it is necessary to define the dimensions of the irradiated area: length ($l$) and width ($w$). Then, it has been considered one of the borders as the origin of the coordinate system ($x = 0$ and $y = 0$). After, that is necessary to define the lamp dimension, length ($L$). Then, the number of lamps ($n$) must be given, as well their position in relation the origin of coordinate system ($X_L$, $Y_L$), its height in relation to the irradiated area ($H$) and the imposed power. With all these parameters, using the final derived equation, the heat flux in the area can be evaluated. When the parameter $X$ in each lamp is used in the equation, its value is $X$ minus, $X_L$, as well for $Y$.

The program to simulate the heat flux from an infrared lamp array has been made in Microsoft Excel, using the VBA programming language.

Figure (8) shows the cells where the user should put the required values, dimensions of the irradiated surface, and dimensions and characteristics of the lamp.

![Figure 8: Cells where the user defines the irradiated surface, and dimension and the lamp characteristics.](image)

Figure (9) shows the cells where the user should put values of the number of lamps, and its inputs: power ($Watts$), height (mm), position ($X,Y$) in relation to the center of the coordinate system (mm, mm).
Figure 9: Cells where the user defines the lamp array parameters: number of lamps \((n)\) and the inputs in each lamp (power, height and the lamp center).

The program, named **IR Lamp**, has three data sheets: data, results, 3D or Graphics. The data sheet is shown above in Fig. (8) and (9). Results are shown as values of heat flux at the irradiated area, which are divided in some points. These points are placed with 50 mm of distance to the next, starting in the origin until the limits of the area (length \(l\) and width \(w\)). For example, with the shown values in Fig. (8) and (9), the results present are in a matrix whose order is \(17 \times 17\). The values in columns are referent to the same position \(X\), varying the value of \(Y\) from 0 up to 800. In line, it can be observed the same behavior; to the same position of \(Y\) the value of \(Y\) is varied since 0 to 800. The format of the result matrix is present in Fig. (10) ahead:

\[
y \setminus x \quad 0 \quad 50 \quad 100 \quad \ldots \quad 800
\]

\[
0 \quad I_{1,1} \quad I_{1,2} \quad I_{1,3} \quad \ldots \quad I_{1,17}
\]

\[
50 \quad I_{2,1} \quad I_{2,2} \quad I_{2,3} \quad \ldots \quad I_{2,17}
\]

\[
100 \quad I_{3,1} \quad I_{3,2} \quad I_{3,3} \quad \ldots \quad I_{3,17}
\]

\[
\vdots \quad \ddots \quad \ddots \quad \ddots \quad \ddots \quad \ddots \quad \ddots
\]

\[
800 \quad I_{17,1} \quad I_{17,2} \quad I_{17,3} \quad \ldots \quad I_{17,17}
\]

Figure 10: Format of the results, to evaluate the heat flux intensity, in each position \(i,j\).

With the value above, Fig. (10), graphics to show the results are in 3D Graphic and 2D Graphic sheets. Using as the maximum value in the scale (value 100), the graphics are presented in Fig. (11) and (12):
5. Conclusions

Using the model cited in this paper, the program IR Lamp presents excellent results to simulate the heat flux from a lamp array. With this program, it is possible to determine the desirable heat flux on a given irradiated area. To obtain this heat flux, the program determines the necessary number of lamps and the configuration of the setup. The results are shown in two graphics, which can be accepted as predicted setup, to design the real test apparatus.

6. Acknowledgements

We would like to thank the CNPq and INPE/LIT. Also, we would like to thank the LIT Thermal Vacuum Group for the support in the manufacturing of the experimental apparatus and carrying out of the tests.

7. References